

Exercise 27

The half-life of palladium-100, ^{100}Pd , is four days. (So half of any given quantity of ^{100}Pd will disintegrate in four days.) The initial mass of a sample is one gram.

- (a) Find the mass that remains after 16 days.
- (b) Find the mass $m(t)$ that remains after t days.
- (c) Find the inverse of this function and explain its meaning.
- (d) When will the mass be reduced to 0.01g?

Solution

After 16 days, four half-lives will have passed, so the mass will be

$$1 \text{ gram} \times \left(\frac{1}{2}\right)^4 = \frac{1}{16} \text{ gram}.$$

The general formula for the mass of palladium-100 after time t is

$$m(t) = 1 \left(\frac{1}{2}\right)^{t/4} = 2^{-t/4}.$$

Solve it for t by taking the natural logarithm of both sides.

$$\ln m(t) = \ln 2^{-t/4}$$

Bring the exponent down in front.

$$\ln m(t) = -\frac{t}{4} \ln 2$$

So the inverse function is

$$t = -\frac{4}{\ln 2} \ln m(t).$$

This gives the time corresponding to how much mass is left. If the mass left is 0.01g, then the time it takes is

$$t = -\frac{4}{\ln 2} \ln 0.01 \approx 26.57 \text{ days}.$$